

Numbers sets

The Natural Numbers ($\mathbb{N}$) = $\{1, 2, 3, \dots\}$

The Whole Numbers ($\mathbb{N}_0$) = $\{0, 1, 2, 3, \dots\}$

The Integer Numbers ($\mathbb{Z}$) = $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

The Rational Numbers ($\mathbb{Q}$) = $\left\{ \frac{a}{b}, \text{ s.t } a, b \in \mathbb{Z}, b \neq 0 \right\}$

The Real Numbers ($\mathbb{R}$) = $(-\infty, \infty)$

The Irrational Numbers ($\mathbb{Q}^c$) = $\mathbb{R} \setminus \mathbb{Q}$

The Complex Numbers ($\mathbb{C}$) = $\{a + ib, \text{ s.t } a, b \in \mathbb{R}, i = \sqrt{-1}\}$

Note that:

$$\mathbb{N} \subseteq \mathbb{N}_0 \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

Some Important and Common Irrational Numbers

1- pi (π):

Pi (π): It is the ratio between the circumference of a circle to its diameter. It is also called Archimedes' constant and approximately equals 3.1415926 .

The number π is an irrational number , meaning that it cannot be expressed exactly as a ratio of two integers, although fractions such as $\frac{22}{7}$ are commonly used to approximate it . Consequently, its decimal representation never ends, nor enters a permanently repeating pattern. It is a transcendental number , meaning that it cannot be a solution of an algebraic equation involving only finite sums, products, powers, and integers. The decimal digits of π appear to be randomly distributed , but no proof of this conjecture has been found.

For thousands of years, mathematicians have attempted to extend their understanding of π , sometimes by computing its value to a high degree of accuracy. Ancient civilizations, including the Egyptians and Babylonians , required fairly accurate approximations of π for practical computations. Around 250 BC, the Greek mathematician Archimedes created an algorithm to approximate π with arbitrary accuracy. In the 5th century AD, Chinese mathematicians approximated π to seven digits,

while Indian mathematicians made a five-digit approximation, both using geometrical techniques. The first computational formula for π , based on infinite series was discovered a millennium later. The earliest known use of the Greek letter π to represent the ratio of a circle's circumference to its diameter was by the Welsh mathematician William Jones in 1706. The invention of calculus soon led to the calculation of hundreds of digits of π , enough for all practical scientific computations. Nevertheless, in the 20th and 21st centuries, mathematicians and computer scientists have pursued new approaches that, when combined with increasing computational power, extended the decimal representation of π to many trillions of digits. These computations are motivated by the development of efficient algorithms to calculate numeric series, as well as the human quest to break records.

The number pi (π) is one of the most important mathematical constants, used in many scientific and engineering fields. Its importance lies in the following :

Geometry and Circles as it represents the ratio between the circumference of any circle and its diameter.

It is used to calculate the area and volume of circular shapes, such as: circle area πr^2 and sphere volume $\frac{4}{3}\pi r^3$, where r is the radius of the circle or the sphere.

Mathematics and Calculus as it appears in integrals and mathematical series, such as the Fourier series for sine (sin) and cosine (cos) functions.

Fourier series :

For a periodic function $f(x)$ with period T, its Fourier series is given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos \frac{2n\pi x}{T} + b_n \sin \frac{2n\pi x}{T} \right]$$

where the coefficients are given by:

$$a_0 = \frac{1}{T} \int_T f(x) dx, \quad a_n = \frac{2}{T} \int_T f(x) \cos \frac{2n\pi x}{T} dx, \quad \text{and} \quad b_n = \frac{2}{T} \int_T f(x) \sin \frac{2n\pi x}{T} dx.$$

Note that a function satisfies that $f(x) = f(x + T)$ is called a periodic function with period T.

It plays a crucial role in Fourier transforms, which are used in signal, sound, and image processing.

Physics and Engineering as it is used in circular motion and dynamics formulas, such as angular velocity: $\omega = 2\pi f$.

In quantum mechanics, it appears in Schrödinger's equation, and in general relativity, it is used to calculate spacetime curvature.

Computer Science and Cryptography as it is applied in random algorithms and cryptography, appearing in probability distributions such as the Gaussian (normal) distribution.

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2}$$
 where $f(x)$ is the probability density function, σ is the standard deviation, and μ is the mean.

Astronomy and Geography as it is used in calculating planetary orbits and the motion of celestial bodies, and it is essential in longitude and latitude measurements and geographic mapping.

We can say that the number π is not just a mathematical constant; it is a fundamental element in science, engineering, and technology, making it one of the most important numbers in the world.

2- e (Euler's number):

It is a mathematical constant approximately equals to 2.71828 and it is the base of the natural logarithm and exponential function.

e is a transcendental number.

The first references to this constant were published in 1618 in the table of an appendix of a work on logarithms by John Napier. However, this did not contain the constant itself, but simply a list of logarithms to the base e. It is assumed that the table was written by William Oughtred. In 1661, Christiaan Huygens studied how to compute logarithms by geometrical methods and calculated a quantity that, in retrospect, is the base-10 logarithm of e, but he did not recognize e itself as a quantity of interest.

The constant itself was introduced by Jacob Bernoulli in 1683, for solving the problem of continuous compounding of interest.

In his solution, the constant e occurs as the limit $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$, where n represents the number of intervals in a year on which the compound interest is evaluated.

The first symbol used for this constant was the letter b by Gottfried Leibniz in letters to Christiaan Huygens in 1690 and 1691.

Leonhard Euler started to use the letter e for the constant in 1727 or 1728, in an unpublished paper on explosive forces in cannons, and in a letter to Christian Goldbach on 25 November 1731. The first appearance of e in a printed publication was in Euler's *Mechanica* (1736). It is unknown why Euler chose the letter e. Although some researchers used the letter c in the subsequent years, the letter e was more common and eventually became standard.

Euler proved that e is the sum of the infinite series

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots, \text{ where } n! \text{ is the factorial of } n.$$

The equivalence of the two characterizations using the limit and the infinite series can be proved via the binomial theorem.

We can find this constant in a lot of applications such as following:

The number e is of great importance in mathematics, alongside 0 , 1 , π , and i . All five appear in one formulation of Euler's identity $e^{i\pi} + 1 = 0$ and play important and recurring roles across mathematics.

Computer Science in algorithms as randomized algorithms, such as Monte Carlo simulations, often use e in probability calculations and in machine learning (Activation Functions) in deep learning, the softmax function (used in classification problems) is based on exponentials:

$$\sigma(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}.$$

Pharmacokinetics (Drug Absorption & Decay) as the concentration of a drug in the bloodstream follows the relation $C(t) = C_0 e^{-kt}$, and this helps in determining dosage schedules.

In Quantum Mechanics (Schrödinger's Equation), the wave function of a quantum system is often expressed using e : $\psi(x, t) = A e^{i(kx - \omega t)}$.

This describes how quantum particles behave as waves. The presence of $e^{i\theta}$ is crucial in solving the Schrödinger equation.

3-The Golden Ratio ϕ (phi) :

It is an irrational number its value is $\frac{1 + \sqrt{5}}{2}$ and approximately equal to 1.6180339 .

It appears in geometry , art , nature , architecture and financial markets.

The golden ratio satisfies the quadratic equation : $x^2 - x - 1 = 0$.

Also the golden ratio appears in Fibonacci sequence as it takes the form 1,1,2,3,5,8,13,21,34,55,89,....

which has a strong connection to the Golden Ratio. The ratio of consecutive Fibonacci

numbers approaches ϕ : $\frac{F_n}{F_{n-1}} \approx 1.618$.

For example: $\frac{89}{55} \approx 1.61818$

Some applications of the Golden Ratio:

In nature the flower petals and seeds: The arrangement of sunflower seeds follows a spiral based on ϕ .

Human body proportions as the ratio of different body parts, such as the forearm to the hand, often follows the Golden Ratio.

Animal patterns, the spiral shells (e.g., Nautilus shell) grow in a logarithmic spiral related to ϕ .

Art and Architecture, the Parthenon in Greece follows Golden Ratio proportions, Leonardo da Vinci's Vitruvian Man and Mona Lisa use ϕ , and many famous paintings and compositions are structured around the Golden Ratio.

Technology and Design, logos like Apple, Twitter, and Google use Golden Ratio proportions, and web design and user interfaces often apply the Golden Rectangle for a visually appealing layout.

Finance and stock markets, the Fibonacci retracement levels in technical analysis use Golden Ratio percentages (23.6%, 38.2%, 61.8%) to predict price movements in stocks and cryptocurrencies.

Answer the following questions:

1- is called the Archimedes' constant.

- a) e **b) π** c) ϕ d) None of them

2- Using the value of pi approximated to two decimal places, the area of a circle with radius 2 is.....

- a) 12.56** b) 12.564 c) 12.566 d) 12.57

3- Which of the following is a periodic function?

- a) $\sin x$** b) x c) x^2 d) $\ln x$

4- We use in quantum mechanics (Schrödinger's Equation)

- a) ϕ b) π c) e **d) both π and e**

5- is a solution for $x^2 - x - 1 = 0$.

- a) ϕ** b) π c) e d) None of them

6- appears in the angular velocity formula.

- a) φ b) π c) e d) None of them

7- is called Euler's identity.

- a) $e^{i\pi} + 1 = 0$ b) $e^{i\pi} - 1 = 0$ c) $e^{i\pi} + 1 = i$ d) $e^{-i\pi} + 1 = 0$

8- The first letter used to refer to Euler's constant is.....

- a) A b) C c) b d) e

9- Which constant (s) appear in the Gaussian (normal) distribution?

- a) φ b) π c) e d) both π and e

10- The arrangement of sunflower seeds follows a spiral based on.....

- a) φ b) π c) e d) both π and e